Q.P. Code:23HS0835

R23

SIDDHARTH INSTITUTE OF ENGINEERING &TECHNOLOGY : PUTTUR
(AUTONOMOUS)

Siddharth Nagar, Narayanavanam Road – 517583

QUESTION BANK (DESCRIPTIVE)

Subject with Code: PROBABILITY AND COMPLEX VARIABLES (**23HS0835**)

Branches: **ECE**

Year & Sem: II-B. Tech & I-Sem **Regulation:** R23

UNIT –I
PROBABILITY & RANDOM VARIABLE

1	a) State the axioms of probability	[L1] [CO1]	**[2M]**
	b) Define a random variable with suitable examples	[L1] [CO1]	**[2M]**
	c) State Baye's theorem	[L1] [CO1]	**[2M]**
	d) Define mean and variance of a probability distribution	[L2] [CO1]	**[2M]**
	e) Write the density function of uniform distribution	[L1] [CO1]	**[2M]**
2	a) A class consists of 6 girls and 10 boys. If a committee of 3 is chosen at random from the class. Find the Probability that (i) 3 boys are selected (ii) exactly 2 girls are selected	[L1] [CO1]	**[5M]**
	b) If three coins are tossed. Find the probability of getting (i) 3 heads (ii) 2 heads (iii) no heads.	[L1] [CO1]	**[5M]**
3	a) Two cards are selected at random from 10 cards numbered 1 to 10. Find the probability that the sum is even if (i) The two cards are drawn together. (ii) The two cards drawn one after other with replacement.	[L5] [CO1]	**[5M]**
	b) Determine (i) $P\left(B/A\right)$ (ii) $P\left(A/B^C\right)$ if A and B are events with $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{4}$, $P(A\cup B)=\frac{1}{2}$.	[L5] [CO1]	**[5M]**
4	a) In a certain town 40% have brown hair, 25% have brown eyes and 15% have both brown hair and brown eyes. A person is selected at random from the town. i) If he has brown hair, what is the probability that he has brown eyes also. ii) If he has brown eyes, determine the probability that he does not have brown hair?	[L1] [CO1]	**[5M]**
	b) The probability that students A,B,C,D solve the problem are $\frac{1}{3}, \frac{2}{5}, \frac{1}{5}$ and $\frac{1}{4}$ respectively if all of them try to solve the problem, what is the probability that the problem is solved.	[L3] [CO1]	**[5M]**
5	Two dice are thrown. Let A be the event that the sum of the points on the faces is 9. Let B be the event that at least one number is 6. Find (i) $P(A\cap B)$ (ii) $P(A\cup B)$ (iii) $P(A^c\cup B^c)$ (iv) $P(A^c\cap B^c)$ (v) $P(A\cap B^c)$	[L1] [CO1]	**[10M]**
6	In a certain college 25% of boys and 10% of girls are studying mathematics. The girls Constitute 60% of the student body. (a) What is the probability that mathematics is being studied? (b) If a student is selected at random and is found to be studying mathematics, find the probability that the student is a girl? (c) a boy	[L1] [CO1]	**[10M]**

<table>
<tr><td>7</td><td>a) A continuous random variable has the probability density function
$f(x) = \begin{cases} kxe^{-\lambda x}, for\ x \ge 0, \lambda > 0 \\ \quad 0, \qquad otherwise \end{cases}$
Determine (i) k (ii) Mean (iii) Variance</td><td>[L5] [CO1]</td><td>[5M]</td></tr>
<tr><td></td><td>(b)A random variable X has the following probability function
<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>P(x)</td><td>0</td><td>K</td><td>2K</td><td>2K</td><td>3K</td><td>K^2</td><td>$2K^2$</td><td>$7K^2+K$</td></tr>
</table>
Determine (i) K (ii) Evaluate $P(X\geq 6)$ and $P(0<X<5)$ (iii) if $P(X\leq K)>1/2$, find the minimum value of K</td><td>[L5] [CO1]</td><td>[5M]</td></tr>
<tr><td>8</td><td>a) Derive mean and variance of Binomial distribution.</td><td>[L1] [CO1]</td><td>[5M]</td></tr>
<tr><td></td><td>b) 20% of items produced from a factory are defective. Find the probability that in a sample of 5 chosen at random
(i) one is defective (ii) $p(1 < x < 4)$</td><td>[L1] [CO1]</td><td>[5M]</td></tr>
<tr><td>9</td><td>a) Derive mean and variance of Poisson distribution.</td><td>[L5] [CO1]</td><td>[5M]</td></tr>
<tr><td></td><td>b) If 2% of light bulbs are defective. Find the probability that (i) 2 defective items (ii) at least 3 defective items (iii) $P(2 < x < 5)$in a sample of 100.</td><td>[L1] [CO1]</td><td>[5M]</td></tr>
<tr><td>10</td><td>In a sample of 1000 cases, the mean of certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal find (i) How many students score between 12 and 15 (ii) How many students score above 18? (iii) How many students score below 18?</td><td>[L3] [CO1]</td><td>[5M]</td></tr>
<tr><td>11</td><td>a) The Rayleigh density function is given by
$f(x) = \begin{cases} xe^{-\frac{x^2}{2}} & x \ge 0 \\ 0 & x < 0 \end{cases}$
a) Prove that f(x) satisfies the properties of the pdf
(i) f(x)≥ 0 for all x and (ii) $\int_{\infty}^{\infty} f(x)dx = 1$
b) Find the distribution function $f(x)$
c) Find $P(0.5 < x \le 2)$</td><td>[L5] [CO1]</td><td>[5M]</td></tr>
<tr><td></td><td>b) Define mean and variance of exponential distribution</td><td>[L1] [CO1]</td><td>[5M]</td></tr>
</table>

UNIT –II
OPERATIONS ON RANDOM VARIABLE

<table>
<tr><td rowspan="5">1</td><td>a) Define moments about origin and mean</td><td>[L1] [CO2]</td><td>[2M]</td></tr>
<tr><td>b) Define variance</td><td>[L1] [CO2]</td><td>[2M]</td></tr>
<tr><td>c) State Chebyshev’s inequality</td><td>[L1] [CO2]</td><td>[2M]</td></tr>
<tr><td>d) Explain interval conditioning</td><td>[L2] [CO2]</td><td>[2M]</td></tr>
<tr><td>e) Explain statistical independence</td><td>[L2] [CO2]</td><td>[2M]</td></tr>
<tr><td rowspan="2">2</td><td>a) The number of customers who visit a car dealer show room on a certain day is a random variable with mean 18 and standard deviation 2.5. with what probability can it be asserted that there will be between 8 and 28 customers</td><td>[L5] [CO2]</td><td>[5M]</td></tr>
<tr><td>b) If $\bar{x}$ is the number appearing on a die when it is thrown show that the Chebyshev’s theorem gives
$P(|\bar{x}-\mu| > 2.5)$<0.47 while the actual probability is zero</td><td>[L5] [CO2]</td><td>[5M]</td></tr>
<tr><td>3</td><td>From the following data
<table>
<tr><td>X</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>P(x)</td><td>0.1</td><td>0.4</td><td>0</td><td>0.2</td><td>0.1</td></tr>
</table>
To find(i) mean (iv) $E(7X \pm 8)$
(ii) variance (v) $E(4X \pm 6)^2$
(iii) E(3x±4) (iv) $V(4X \pm 6)$</td><td>[L5] [CO2]</td><td>[10M]</td></tr>
<tr><td>4</td><td>The continuous density function of a random variable X is defined as
$$f(x) = \begin{cases} \frac{x}{2} & 0 \le x \le 1 \\ \frac{1}{2} & 1 \le x \le 2 \\ \frac{1}{2}(3-x) & 2 \le x \le 3 \end{cases}$$
To find (i) mean (ii) mean square (iv) E[X^3] (iv) variance</td><td>[L1] [CO2]</td><td>[10M]</td></tr>
<tr><td>5</td><td>For the Random variable X whose density function of continuous random variable X
$$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & otherwise \end{cases}$$
(i) Find Moment generating function
(ii) Mean and variance</td><td>[L1] [CO2]</td><td>[10M]</td></tr>
<tr><td>6</td><td>Find the first moment of exponential probability density function of continuous random variable X given
$$f(x) = \begin{cases} \frac{1}{b} e^{-\left(\frac{x-a}{b}\right)} & x \ge a \\ 0 & x < a \end{cases}$$
using characteristic function.</td><td>[L1] [CO2]</td><td>[10M]</td></tr>
<tr><td rowspan="2">7</td><td>a) Define moment generating function</td><td>[L5] [CO2]</td><td>[2M]</td></tr>
<tr><td>b) find mean and variance of binomial distribution using mgf</td><td>[L5] [CO2]</td><td>[8M]</td></tr>
<tr><td rowspan="2">8</td><td>a) Define characteristic function</td><td>[L5] [CO2]</td><td>[2M]</td></tr>
<tr><td>b) find mean and variance of poisson distribution using cf</td><td>[L5] [CO2]</td><td>[8M]</td></tr>
<tr><td>9</td><td>Given the function
$$f(x,y) = \begin{cases} b(x+y)^2 & -2 < x < 2, -3 < y < 3 \\ 0 & elsewhere \end{cases}$$
(i)find a constant ‘b’ such that this is a valid density function
(ii)determine the marginal density function $f(x) and\ f(y)$</td><td>[L1] [CO2]</td><td>[10M]</td></tr>
</table>

<table>
<tr>
<td>10</td>
<td>For the following probability distribution of X and Y
Find (i) $P(X\leq1, Y=2)$ (ii) $P(X\leq1)$ (iii) $P(Y=3)$ (iv) $P(Y\leq3)$
(v) $P(x<4, y\leq4)$ also find the marginal distribution X and Y
<table>
<tr><td>Y
X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>1/32</td><td>2/32</td><td>2/32</td><td>3/32</td></tr>
<tr><td>1</td><td>1/16</td><td>1/16</td><td>1/8</td><td>1/8</td><td>1/8</td><td>1/8</td></tr>
<tr><td>2</td><td>1/32</td><td>1/32</td><td>1/64</td><td>1/64</td><td>0</td><td>2/64</td></tr>
</table>
</td>
<td>[L5] [CO2]</td>
<td>[10M]</td>
</tr>
<tr>
<td>11</td>
<td>Let X and Y be jointly continuous random variables with joint density function
$f(x,y) = xy.e^{-(\frac{x^2+y^2}{2})}$ for x>0,y>0
a) Check whether X and Y are independent
b) Find $p(x\leq1, y\leq1)$</td>
<td>[L5] [CO2]</td>
<td>[10M]</td>
</tr>
</table>

UNIT –III
OPERATIONS ON MUTIPLE RANDOM VARIABLES

<table>
<tr><td>1</td><td>a) Define joint characteristic function.</td><td>[L1] [CO3]</td><td>[2M]</td></tr>
<tr><td></td><td>b)Define and explain conditional probability mass function.</td><td>[L1] [CO3]</td><td>[2M]</td></tr>
<tr><td></td><td>c) The correlation coefficient of two random variable X and Y is -1/4 while their variances are 3 and 5. Find the covariance.</td><td>[L1] [CO4]</td><td>[2M]</td></tr>
<tr><td></td><td>d) When random variables are called jointly Gaussian.</td><td>[L2] [CO4]</td><td>[2M]</td></tr>
<tr><td></td><td>e) List some properties of Gaussian random variable.</td><td>[L2] [CO4]</td><td>[2M]</td></tr>
<tr><td>2</td><td>The joint probability function of two discrete random variable X and Y is given by
$f(x,y) = cxy\ for\ x = 1,2,3\ and\ y = 1,2,3$
and equals zero otherwise. Find
a) constant c d) P (X≤ 2)
b) $P(x = 2, y = 3)$ e) $P(y < 2)$
c)$P(1 \leq x \leq 2, y \leq 2)$ f)$P(x = 1)$ g)$P(y = 3)$</td><td>[L5] [CO3]</td><td>[10M]</td></tr>
<tr><td>3</td><td>X and Y are independent random variable each having density function of the form
$f(u) = 2.e^{-2u}\ for\ u \geq 0$
0 otherwise
Find (a) $E(X + Y)$ (b) $E(X^2 + Y^2)$ (c) $E(XY)$</td><td>[L5] [CO3]</td><td>[10M]</td></tr>
<tr><td>4</td><td>The joint space for two random variable X and Y and corresponding probabilities are shown in table. Find
(a) F(x,y) (b)marginal distribution function of x and y
(c) find P$(0.5 < x < 1.5)$ (d) find $P(x \leq 1, y \leq 2)$ and
(e) find $P(1 < x \leq 2, y \leq 3)$
<table>
<tr><td>X, Y</td><td>1,1</td><td>2,2</td><td>3,3</td><td>4,4</td></tr>
<tr><td>P</td><td>0.05</td><td>0.35</td><td>0.45</td><td>0.15</td></tr>
</table></td><td>[L1] [CO3]</td><td>[10M]</td></tr>
<tr><td>5</td><td>If the function
$$f(x,y) = \begin{cases} be^{-2x}\ \cos\left(\frac{y}{2}\right) & 0 \leq x \leq 1 \\ 0 & 0 \leq y \leq \pi \\ 0 & elsewhere \end{cases}$$
Where ‘b’ is a positive constant is a valid joint density function, find ‘b’</td><td>[L1] [CO3]</td><td>[10M]</td></tr>
<tr><td>6</td><td>The joint density function of the random variables X and Y is given as $f(x,y) = 8xy\quad for\ \ 0 \leq x \leq 1,\ o \leq y \leq x$
Find (a) marginal density of x (b) marginal density of y
(c) conditional density of x (d) conditional density of y</td><td>[L1] [CO4]</td><td>[10M]</td></tr>
<tr><td>7</td><td>Given the function $f(x,y) = \begin{cases}\frac{x^2+y^2}{8\pi}\ , x^2 + y^2 < b\end{cases}$
(a)find the constant ‘b’ so that this is a valid joint density function
(b) find $p(0.5b < x^2 + y^2 < 0.8b)$</td><td>[L1] [CO4]</td><td>[10M]</td></tr>
<tr><td>8</td><td>The density function of two random variable X and Y is
$$f(x,y) = 16.e^{-4(x+y)}$$
Find the mean of the function
$g(X,Y) = 5 \quad for\ 0 < x \leq \frac{1}{2}; 0 < Y \leq \frac{1}{2}$
$= -1 \quad for \frac{1}{2} < X\ and\ for \frac{1}{2} < y$
$= 0\ for\ all\ other\ X\ and\ Y$</td><td>[L1] [CO4]</td><td>[10M]</td></tr>
</table>

<table>
<tr>
<td>9</td>
<td>Two random variables X and Y have means 1 and 2 respectively and variance 4 and 1 respectively. Their correlation coefficient is 0.4.new the random variable W and V are defined as
V= -X+2Y ; W=X+3Y
Find the
(a) Means (b)variance (c) correlations
(d) correlation coefficient of V and W</td>
<td>[L1] [CO4]</td>
<td>[10M]</td>
</tr>
<tr>
<td>10</td>
<td>Calculate the coefficient of variation between X and Y from the following data
<table>
<tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>Y</td><td>9</td><td>8</td><td>10</td><td>12</td><td>11</td><td>13</td><td>14</td><td>16</td><td>15</td></tr>
</table>
</td>
<td>[L5] [CO4]</td>
<td>[10M]</td>
</tr>
<tr>
<td>11</td>
<td>Let X and Y be the random variables having joint density function
$f(x,y) = x + y \quad for\ o \leq x \leq 1, 0 \leq y \leq 1$
0 Otherwise
Find (a) var(X) (b) var(Y) (C)$\sigma_{x\,y}$ (d) ρ</td>
<td>[L5] [CO4]</td>
<td>[10M]</td>
</tr>
</table>

UNIT –IV
COMPLEX VARIABLE – DIFFERENTIATION

1	a) Define analytic function.	[L1] [CO5]	**[2M]**
	b) State Cauchy-Riemann (C-R) equations in cartesian coordinates.	[L1] [CO5]	**[2M]**
	c) Find where the function $w = \frac{1}{z}$ ceases to be analytic.	[L1] [CO5]	**[2M]**
	d) Define harmonic function.	[L1] [CO5]	**[2M]**
	e) Prove that $f(z) = \bar{z}$ is not an analytic at any point.	[L5] [CO5]	**[2M]**
2	a) Show that z^2 is an analytic for all z.	[L2] [CO5]	**[5M]**
	b) Determine p such that the function $f(z) = \frac{1}{2}log(x^2 + y^2) + i\,tan^{-1}\left(\frac{px}{y}\right)$ is an analytic function.	[L5] [CO5]	**[5M]**
3	a) Find whether $f(z) = sinxsiny - icosxcosy$ is an analytic or not.	[L1] [CO5]	**[5M]**
	b) Determine whether the function $f(z) = 2xy + i(x^2 - y^2)$ is analytic.	[L5] [CO5]	**[5M]**
4	a) Show that $\mathrm{u} = \mathrm{e}^{-\mathrm{x}}(\mathrm{x}\sin\mathrm{y} - \mathrm{ycosy})$ is harmonic function.	[L2] [CO5]	**[5M]**
	b) Show that $u = 2\log(x^2 + y^2)$ is harmonic function.	[L2] [CO5]	**[5M]**
5	Verify that $u = x^2 - y^2 - y$ is harmonic in the whole complex plane and find a conjugate harmonic function v of u?	[L4] [CO5]	**[10M]**
6	Prove that, if $u = x^2 - y^2 : v = \frac{-y}{x^2+y^2}$ both u and v satisfy Laplace's equation, but $u + iv$ is not a analytic function.	[L5] [CO5]	**[10M]**
7	Show that i) $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \bar{z}}$. ii) $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\|f(z)\|^2 = 4\|f'(z)\|^2$ where $f(z)$ is an analytic function.	[L1] [CO5]	**[10M]**
8	Prove that the function f(z) defined by $f(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, (z \neq 0)\ \ and\ f(z) = 0\,, (z = 0)$ is continuous and the Cauchy-Riemann equations are satisfied at origin.	[L5] [CO5]	**[10M]**
9	Find 'a' and 'b' if $f(z) = (x^2 - 2xy + ay^2) + i(bx^2 - y^2 + 2xy)$ is analytic. Hence find f(z) interms of z.	[L1] [CO5]	**[10M]**
10	a) Determine the analytic function whose real part is e^{x}cosy.	[L5] [CO5]	**[5M]**
	b) Find the analytic function whose imaginary is $\frac{x-y}{x^2+y^2}$.	[L1] [CO5]	**[5M]**
11	a) Find the analytic function f(z) interms of z whose real part is $\mathrm{x}^3 - 3\mathrm{xy}^2$.	[L1] [CO5]	**[5M]**
	b) Find the analytic function whose imaginary part is $\mathrm{e}^{\mathrm{x}}(\mathrm{x}\sin\mathrm{y} + \mathrm{ycosy})$.	[L1] [CO5]	**[5M]**

UNIT –V
COMPLEX VARIABLE – INTEGRATION

1	a) Define Line integral	[L1] [CO6]	**[2M]**
	b) State Cauchy's integral theorem.	[L1] [CO6]	**[2M]**
	c) State Cauchy Integral formula.	[L1] [CO6]	**[2M]**
	d) Expand e^z as Taylor's series in powers of (z-3).	[L2] [CO6]	**[2M]**
	e) State Cauchy Residue theorem.	[L1] [CO6]	**[2M]**
2	a) Evaluate $\int_{(0,0)}^{(1,3)} 3x^2ydx + (x^3 - 3y^2)dy$ along the curve $y = 3x$.	[L5] [CO6]	**[5M]**
	b) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path $y = x$.	[L5] [CO6]	**[5M]**
3	Evaluate $\int_0^{3+i} z^2 dz$, i) along the line $y = \frac{x}{3}$ ii) along the parabola $x = 3y^2$.	[L5] [CO6]	**[10M]**
4	Show that $\int_c (z+1)dz = 0$ where 'c' is the boundary of the square whose vertices at the points $z = 0, z = 1, z = 1 + i, z = i$.	[L1] [CO6]	**[10M]**
5	a) Evaluate $\int_c \frac{e^{2z}}{(z-1)(z-2)}dz$ where 'c' is the circle $\|z\| = 3$.	[L5] [CO6]	**[5M]**
	b) Evaluate $\int_c \frac{e^z}{(z-1)(z-4)}dz$ where 'c' is $\|z\| = 2$.	[L5] [CO6]	**[5M]**
6	Evaluate $\int_c \frac{z-3}{z^2+2z+5}dz$ where 'c' is the circle i) $\|z\| = 1$ ii) $\|z + 1 - i\| = 2$	[L5] [CO6]	**[10M]**
7	a) Expand $f(z) = sinz$ in Taylor's expansion of in powers of $\left(z - \frac{\pi}{4}\right)$.	[L2] [CO6]	**[5M]**
	b) Expand $f(z) = \frac{1}{z^2-z-6}$ in Taylor's series about $i)\ z = -1\ ii)\ z = 1$.	[L2] [CO6]	**[5M]**
8	a) Find the Laurent expansion of $\frac{1}{z^2-4z+3}$ for i) $1 < \|z\| < 3$ ii) $\|z\| < 1$.	[L1] [CO6]	**[5M]**
	b) Determine the poles of the function i) $\frac{z}{\cos z}$ ii) cotz.	[L5] [CO6]	**[5M]**
9	a) Find the poles of the function $f(z) = \frac{1}{(z+1)(z+3)}$ and residues at each poles.	[L1] [CO6]	**[5M]**
	b) Find the residue of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ at each pole.	[L1] [CO6]	**[5M]**
10	Evaluate $\oint_c \frac{4-3z}{z(z-1)(z-2)}dz$ where 'c' is circle $\|z\| = \frac{3}{2}$ using residue theorem.	[L5] [CO6]	**[10M]**
11	a) Evaluate $\oint_c \frac{z-3}{z^2+2z+5}dz$ where 'c' is circle given by $\|z + 1 + i\| = 2$ using Residue theorem.	[L5] [CO6]	**[5M]**
	b) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ using residue theorem	[L5] [CO6]	**[5M]**